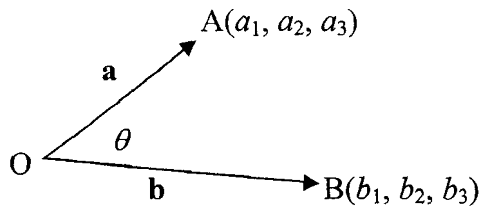
CALCULUS 3
Q101

(3D-SPACE)
$\Re^3$

Vector Basics: Dot Product Summary (12.3)



$\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, θ is the angle between **a** and **b** $(0 \le \theta \le 2\pi)$

THM: $a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}|\cos\theta$ [Prove using the Law of Cosines]

DEFN: $a_1 b_1 + a_2 b_2 + a_3 b_3 = \mathbf{a} \cdot \mathbf{b}$

THM (after substitution): $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$

COR: $\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

THM: Vectors **a** and **b** are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$

The work done by a constant force **a** as its point of application moves along the vector **b** is $\mathbf{a} \cdot \mathbf{b}$
$W = \mathbf{a} \cdot \mathbf{b}$

<u>Scalar projection of **b** onto **a**</u>:

The magnitude of the force from **b** being applied along **a** is $comp_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|}$

<u>Vector projection of **b** onto **a**</u>:

The vector representation of the force from **b** being applied along **a** is $proj_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}$

Vector Basics: Cross Product Summary (12.4)

DEFN: Vector (Cross) Product: $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

THM: The vector $\mathbf{a}\times\mathbf{b}$ is orthogonal to both **a** and **b**.

Geometric Application: The vector $\mathbf{a}\times\mathbf{b}$ is normal to the plane containing both **a** and **b**.

THM: $|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$

Geometric Application: $|\mathbf{a}\times\mathbf{b}|$, the magnitude of vector $\mathbf{a}\times\mathbf{b}$, is the area of the parallelogram determined by **a** and **b**.

THM: Vectors **a** and **b** are parallel if and only if $\mathbf{a}\times\mathbf{b} = 0$

R^2 SPACE – Rectangular and Polar Coordinate Relationships

$x = r\cos\theta \quad y = r\sin\theta \quad x^2 + y^2 = r^2$

$\Rightarrow \tan\theta = \dfrac{y}{x}$ and $\cos\theta = \dfrac{x}{r}$

(let $r \geq 0$ and $0 \leq \theta \leq 2\pi$)

R^2 SPACE – Quadratic Relationships

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ ellipse

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ hyperbola

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = -1$ nothing

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 0$ dot at origin

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 0$ two intersecting lines

$\Re^3$ — SPACE

I. Points

A. Rectangular Coordinates
B. Cylindrical Coordinates
C. Spherical Coordinates

II. Lines

A. Parametric Equations
B. Vector Valued Functions $f : U \subset R^1 \mapsto R^3$

III. Surface Equations and Graphs

A. Planes
B. Quadratic Surfaces
C. Quadratic Functions $f : U \subset R^2 \mapsto R^1$
D. Cylinders

LESSON 1 – POINTS IN R^3 (15.7, 15.8)

☐ Relationship between Rectangular and Cylindrical Coordinates:

Goal: $(x, y, z) \to (r, \theta, z)$

Thus: $x = r\cos\theta \quad y = r\sin\theta \quad z = z$

(same as before): $\tan\theta = \frac{y}{x} \quad \cos\theta = \frac{x}{r}$

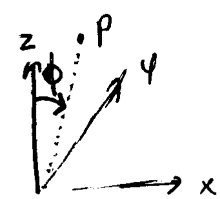
$r \geq 0 \quad 0 \leq \theta < 2\pi$

☐ Relationship between Rectangular and Spherical Coordinates:

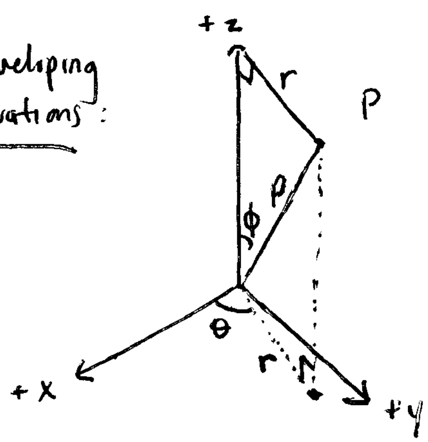
Goal: $(x, y, z) \to (\rho, \theta, \phi)$

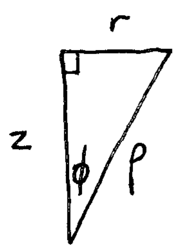
rho theta phi

NOTE that radial distance becomes rho and 3D angle ϕ measured from normal



Developing equations:





Trig relationships:

$\cos\phi = \frac{z}{\rho} \to \boxed{z = \rho\cos\phi}$

$\sin\phi = \frac{r}{\rho} \to \underbrace{r = \rho\sin\phi}$

Since $x = r\cos\theta$ and $y = r\sin\theta$:

$\boxed{x = \rho\cos\theta\sin\phi}$ and $\boxed{y = \rho\sin\theta\sin\phi}$

$\boxed{\begin{array}{l}\text{* NOTE:} \\ \rho \geq 0 \\ 0 \leq \theta < 2\pi \\ 0 \leq \theta \leq \pi\end{array}}$

Limit to π because after π it's the same

use to convert rectangular to spherical

3D Pythagorean: $\boxed{x^2 + y^2 + z^2 = \rho^2}$

Also: $\boxed{\cos\phi = \frac{z}{\rho}}$ $\boxed{\cos\theta = \frac{x}{\rho\sin\phi}}$

LESSON 1 – EXAMPLES

1. Change the cylindrical coordinates $(1,\pi,e)$ and $(1,3\pi/2,5)$ into rectangular coordinates.

(a) $(1,\pi,e)$

(r,θ,z)

$$\left.\begin{array}{ll} x = r\cos\theta \rightarrow & x = \cos\pi = -1 \\ y = r\sin\theta \rightarrow & y = \sin\pi = 0 \\ z = z & z = e \end{array}\right\} (-1,0,e)$$

(b) $(1, 3\pi/2, 5)$

(r, θ, z)

$$\left.\begin{array}{ll} x = r\cos\theta \rightarrow & x = \cos\frac{3\pi}{2} = 0 \\ y = r\sin\theta \rightarrow & y = \sin\frac{3\pi}{2} = -1 \\ z = z & z = 5 \end{array}\right\} (0,-1,5)$$

2. Change the rectangular coordinates $\left(2\sqrt{3},2,-1\right)$ and $\left(4,-3,2\right)$ into cylindrical coordinates.

(a) $(2\sqrt{3}, 2, -1)$

Note the θ'

$r^2 = x^2 + y^2$

$r^2 = 12 + 4 = 16$

$r = 4$

$\cos\theta = \frac{x}{r} \rightarrow \cos\theta' = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$\theta' = \arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$\underbrace{\theta = \theta'}_{\text{true in 1st quadrant}} = \frac{\pi}{6}$

Answer: $\boxed{\left(4, \frac{\pi}{6}, -1\right)}$

(b) $(4, -3, 2) \rightarrow$ ** 4th quadrant **

$r^2 = x^2 + y^2$

$r = 5$

$\cos\theta' = \frac{x}{r} = \frac{4}{5}$ and $\theta' = \arccos\left(\frac{4}{5}\right)$

Now: $\arccos\left(\frac{4}{5}\right) \approx 0.6$ in the 1st quadrant, so to get the 4th quadrant angle,

$\theta = 2\pi - \theta' = 2\pi - \arccos\left(\frac{4}{5}\right)$

Answer: $\left(5, 2\pi - \arccos\left(\frac{4}{5}\right), 2\right)$

Note for cosine, $\theta = \theta'$ in QI and QII $\theta = 2\pi - \theta'$ in QIII and QIV

3. Change the spherical coordinates $(5, \pi/2, \pi)$ and $(4, \pi/3, 3\pi/4)$ into rectangular coordinates.

(a) $\left(5, \frac{\pi}{2}, \pi\right)$

(ρ, θ, ϕ)

$$\left.\begin{aligned} x &= \rho \cos\theta \sin\phi = 5 \cos\tfrac{\pi}{2} \sin\pi = 0 \\ y &= \rho \sin\theta \sin\phi = 5 \sin\tfrac{\pi}{2} \sin\pi = 0 \\ z &= \rho \cos\phi = 5 \cos\pi = -5 \end{aligned}\right\} (0, 0, -5)$$

(b) $\left(4, \frac{\pi}{3}, \frac{3\pi}{4}\right)$

(ρ, θ, ϕ)

$$\left.\begin{aligned} x &= \rho \cos\theta \sin\phi = 4 \cos\tfrac{\pi}{3} \sin\tfrac{3\pi}{4} = \sqrt{2} \\ y &= \rho \sin\theta \sin\phi = 4 \sin\tfrac{\pi}{3} \sin\tfrac{3\pi}{4} = \sqrt{6} \\ z &= \rho \cos\phi = 4 \cos\tfrac{3\pi}{4} = 2\sqrt{2} \end{aligned}\right\} (\sqrt{2}, \sqrt{6}, 2\sqrt{2})$$

4. Change the rectangular coordinates A. $(0,\sqrt{3},1)$ B. $(-1,1,\sqrt{6})$ C. $(1,-1,\sqrt{6})$ D. $(-1,-1,-\sqrt{6})$ into spherical coordinates.

(a) $(0,\sqrt{3},1)$

Note θ'

$\rho^2 = 0^2 + \sqrt{3}^2 + 1^2$ $\quad \cos\phi = \frac{z}{\rho} = \frac{1}{2}$ $\quad \cos\theta' = \frac{x}{\rho\sin\phi} = \frac{0}{2\sin\sqrt{3}} = 0$

$\rho = 2$ $\quad \phi = \arccos\frac{1}{2} = \frac{\pi}{3}$ $\quad \theta' = \arccos 0 = \frac{\pi}{2} = \theta$

Answer: $\left(2, \frac{\pi}{2}, \frac{\pi}{3}\right)$

(b) $(-1,1,\sqrt{6})$

Note θ'

$\rho^2 = 1+1+6 = 8$ $\quad \cos\phi = \frac{z}{\rho} = \frac{\sqrt{6}}{2\sqrt{2}} = \frac{\sqrt{3}}{2}$ $\quad \cos\theta' = \frac{x}{\rho\sin\phi} = \frac{-1}{2\sqrt{2}\sin\frac{\pi}{6}}$

$\rho = 2\sqrt{2}$ $\quad \phi = \arccos\frac{\sqrt{3}}{2} = \frac{\pi}{6}$ $\quad \theta' = \arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4} = \theta$

(because quadrant 2)

Answer: $\left(2\sqrt{2}, \frac{3\pi}{4}, \frac{\pi}{6}\right)$

(d) $(-1,-1,-\sqrt{6})$

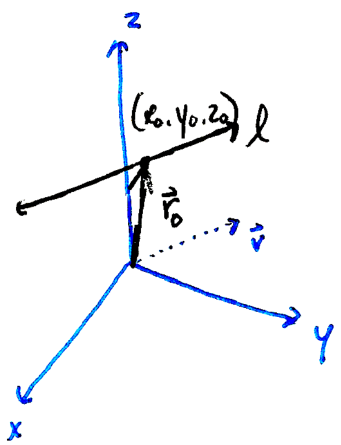
$\rho^2 = 8 \rightarrow \rho = 2\sqrt{2}$ $\quad \cos\phi = \frac{z}{\rho} = \frac{-\sqrt{6}}{2\sqrt{2}} = -\frac{\sqrt{3}}{2}$ $\quad \cos\theta' = \frac{x}{\rho\sin\phi} = \frac{-1}{2\sqrt{2}\sin\frac{5\pi}{6}}$

$\phi = \arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ $\quad \theta' = \arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

$\theta = 2\pi - \theta' = \frac{5\pi}{4}$ (QIII)

Answer: $\left(2\sqrt{2}, \frac{5\pi}{4}, \frac{5\pi}{6}\right)$

LESSON 2 (12.5)
Parametric and Vector Equations for a Line



parametric $\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases}$

Vector equation : $f : R^1 \longmapsto R^3$

$$\left.\begin{aligned} \vec{r}(t) &= \vec{r}_0 + t\vec{v} \\ \vec{r}(t) &= \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle \\ \vec{r}(t) &= \langle x_0 + at,\ y_0 + bt,\ z_0 + ct \rangle \end{aligned}\right\} \quad t \in R$$

Example : eqn of line passing through $(-2, 1, 6)$ parallel to $\langle 7, 8, 9 \rangle$

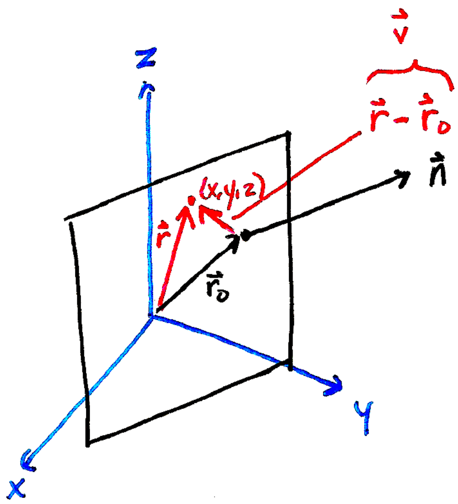
$\hookrightarrow \vec{r}(t) = \langle -2 + 7t,\ 1 + 8t,\ 6 + 9t \rangle$

Symmetric equations : solve for t

$\hookrightarrow \dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

LESSON 2 (12.5)

Equation of a Plane



Plane obtained by considering all vectors $\vec{v}$ such that $\vec{v} \cdot \vec{n} = 0$

$\rightarrow$ that is : $(\vec{r} - \vec{r}_0) \cdot \vec{n} = 0$

$\vec{r} = \langle x, y, z \rangle$

$\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$

$\vec{n} = \langle a, b, c \rangle$ (normal vector)

$$(\vec{r} - \vec{r}_0) \cdot \vec{n} = 0$$

$$(\langle x, y, z \rangle - \langle x_0, y_0, z_0 \rangle) \cdot \langle a, b, c \rangle = 0$$

$$\langle x - x_0, y - y_0, z - z_0 \rangle \cdot \langle a, b, c \rangle = 0$$

$$\boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$$

equivalent to
$\vec{n} = \langle a, b, c \rangle$
(found by the cross product of two vectors on the plane)

EX1: Write a vector normal to the plane $2x-3y+z=6$.

$ax+by+cz=6$

$$\boxed{\vec{n}=\langle 2,-3,1\rangle}$$

EX2: Write the equation of the plane through P(1, 2, 3), Q(-2, 1, 0) and R(5, 1, 2).

Find two vectors:

$\vec{PQ}=\langle -3,-1,-3\rangle$

$\vec{PR}=\langle 4,-1,-1\rangle$

Cross product:

$\vec{n}=\vec{PQ}\times\vec{PR}=\langle -3,-1,-3\rangle\times\langle 4,-1,-1\rangle$

$$=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -3 & -1 & -3\\ 4 & -1 & -1\end{vmatrix}=\begin{vmatrix}-1 & -3\\ -1 & -1\end{vmatrix}\vec{i}-\begin{vmatrix}-3 & -3\\ 4 & -1\end{vmatrix}\vec{j}+\begin{vmatrix}-3 & -1\\ 4 & -1\end{vmatrix}\vec{k}$$

$$=-2\vec{i}-15\vec{j}+7\vec{k}=\langle -2,-15,7\rangle$$

Equation, with $\vec{n}=\langle a,b,c\rangle=\langle -2,-15,7\rangle$ and $(x_0,y_0,z_0)=(1,2,3)$:

$-2(x-1)-15(y-2)+7(z-3)=0$

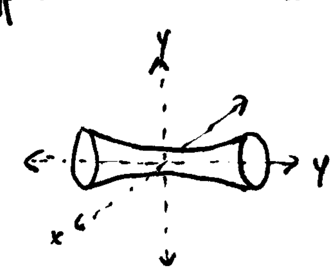
$-2x-15y+7z+2+30-21=0$

$$\boxed{-2x-15y+7z=-11}$$

Example1: Sketch, identify, and describe $16x^2 - 9y^2 + 36z^2 = 144$

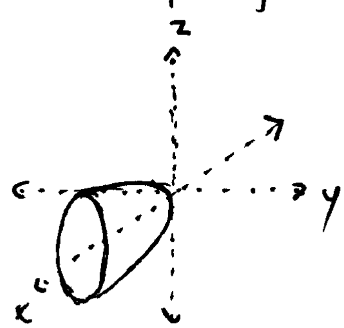
Simplify: $\frac{x^2}{9} - \frac{y^2}{16} + \frac{z^2}{4} = 1$

Hyperboloid of one sheet, travelling the y



Example2: Sketch, identify, and describe $y^2 + 4z^2 = x$. $x = f(y,z)$

Paraboloid opening in x, Vertex (0,0,0) ← always give vertex for "opening in"



Example3: Sketch, identify, and describe $z = 2 - 3x^2 - y^2$

Manipulate: $z = 2 - (3x^2 + y^2)$

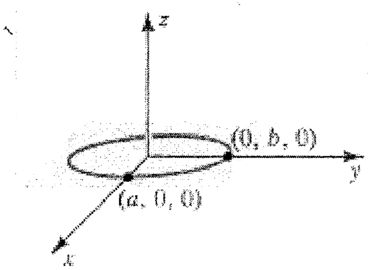
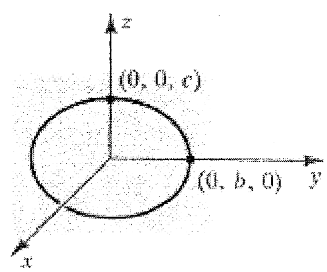
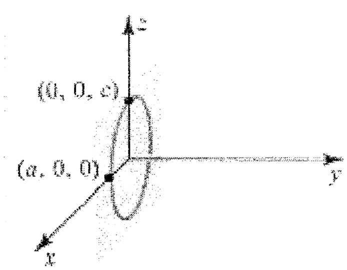
Paraboloid opens in $-z$, vertex $(0,0,2)$

LESSON 3 (12.6)

Quadratic Surfaces

1. $$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1}$$ all terms positive

NAME OF SHAPE: ellipsoid (egg)

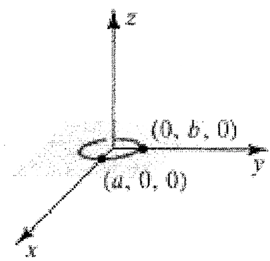
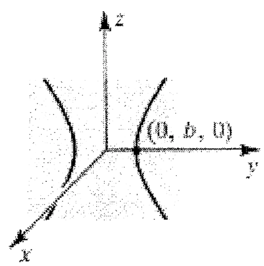
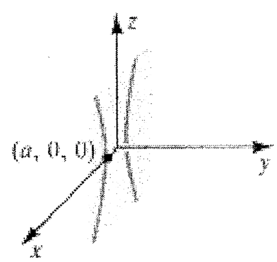
Trace	Equation of trace	Description of trace	Sketch of trace
xy-trace $z=0$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	Ellipse	
yz-trace $x=0$	$\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Ellipse	
xz-trace $y=0$	$\frac{x^2}{a^2}+\frac{z^2}{c^2}=1$	Ellipse	

Ex: $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{16}=1$

$C(0,0,0)$ $r_x=2$ $r_y=3$ $r_z=4$

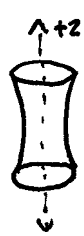
2. $\boxed{\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-\dfrac{z^2}{c^2}=1}$

NAME OF SHAPE: hyperboloid of one sheet

Trace	Equation of trace	Description of trace	Sketch of trace
xy-trace	$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$	Ellipse	
yz-trace	$\dfrac{y^2}{b^2}-\dfrac{z^2}{c^2}=1$	Hyperbola	
xz-trace	$\dfrac{x^2}{a^2}-\dfrac{z^2}{c^2}=1$	Hyperbola	

Ex: $x^2+y^2-z^2=1$

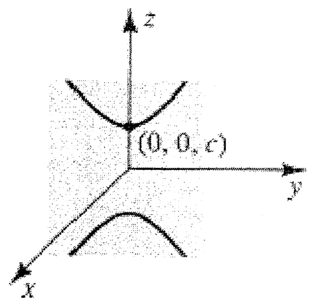
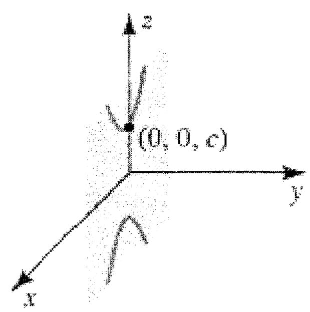
matches the negative term



"tunnels the z-axis"

3. $\boxed{-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1}$

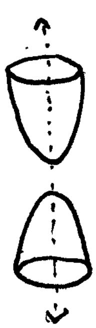
NAME OF SHAPE: hyperboloid of two sheets

Trace	Equation of trace	Description of trace	Sketch of trace
xy-trace	$-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	None	No graph
yz-trace	$-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Hyperbola	
xz-trace	$-\frac{x^2}{a^2}+\frac{z^2}{c^2}=1$	Hyperbola	

Ex: $-x^2-y^2+z^2=1$

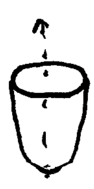
matches the positive term

"opening in the z-direction"



Ex: $z=\sqrt{1+x^2+y^2}$

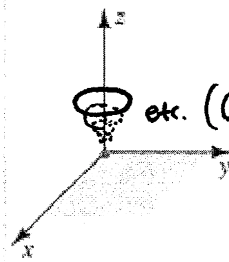
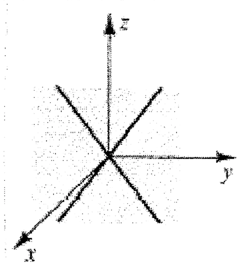
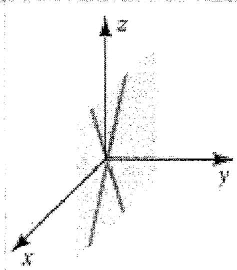
only positive values



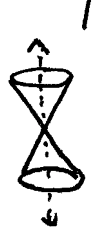
"half hyperboloid of two sheets"

4. $\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0}$ ← equals zero

NAME OF SHAPE: Cone (not two cones)

Trace	Equation of trace	Description of trace	Sketch of trace
xy-trace	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=0$	Origin	 etc. (Circles as z moves)
yz-trace	$\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$	Two intersecting lines	
xz-trace	$\frac{x^2}{a^2}-\frac{z^2}{c^2}=0$	Two intersecting lines	

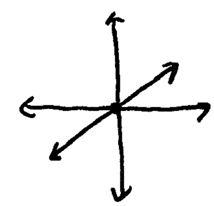
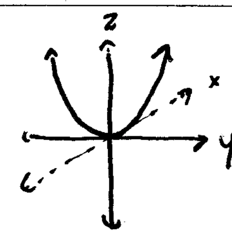
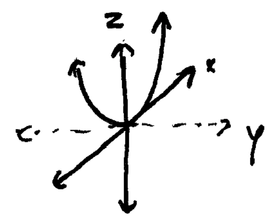
Ex: $x^2+y^2-z^2=0 \Rightarrow$ same as $-x^2-y^2+z^2=0$ (multiply by -1)



"opens in z"
(again, can tell with odd one out)

5. $\boxed{cz = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}}$ $\qquad c > 0$

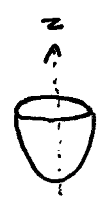
NAME OF SHAPE: paraboloid → is a function

Trace	Equation of trace	Description of trace	Sketch of trace
xy	$0 = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$	point @ origin	
yz	$cz = \dfrac{y^2}{b^2}$	parabola	
xz	$cz = \dfrac{x^2}{a^2}$	parabola	

matches 1st degree term

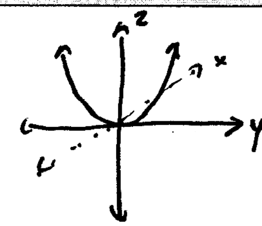
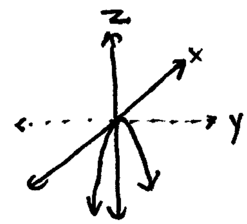
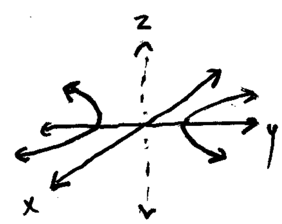
Ex: $z = x^2 + y^2$ "opens in z"

$f(x,y) = x^2 + y^2$ Vertex $(0,0,0)$



6. $cz = \dfrac{y^2}{a^2} - \dfrac{x^2}{b^2}$ $\quad c > 0$

NAME OF SHAPE: hyperbolic parabolloid (saddle)

Trace	Equation of trace	Description of trace	Sketch of trace
yz	$cz = \frac{y^2}{a^2}$	parabola	
xz	$cz = -\frac{x^2}{b^2}$	parabola	
z = 2	$2k = \frac{y^2}{a^2} - \frac{x^2}{b^2}$	hyperbola	

Ex: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = cz \quad c > 0$

Straddling the y   top head pointing in +z

Ex: $z^2 - y^2 - x = 0 \Rightarrow x = z^2 - y^2$ Straddle z top head + x

$-x = y^2 - z^2$ Straddle y top head −x

MATCHING:

916

Exer. 9–20: Match each graph with one of the equations.

A. $\frac{x^2}{25}+\frac{y^2}{9}+\frac{z^2}{4}=1$

B. $x=z^2+\frac{y^2}{4}$

C. $y^2+z^2-x^2=1$

D. $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{4}=0$

E. $z=\frac{x^2}{9}-\frac{y^2}{4}$

F. $z^2-\frac{x^2}{4}-y^2=1$

G. $\frac{z^2}{9}+\frac{y^2}{4}-\frac{x^2}{4}=0$

H. $\frac{x^2}{4}-y^2-z^2=1$

I. $y=\frac{x^2}{4}-\frac{z^2}{9}$

J. $x^2+\frac{y^2}{4}+\frac{z^2}{16}=1$

K. $z=\frac{x^2}{9}+y^2$

L. $\frac{x^2}{4}+\frac{y^2}{16}+\frac{z^2}{9}=1$

M. $y=\frac{z^2}{9}-\frac{x^2}{4}$

N. $y=\frac{x^2}{4}+\frac{z^2}{4}$

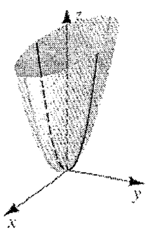
O. $z^2+\frac{x^2}{4}-y^2=1$

P. $\frac{x^2}{4}+\frac{z^2}{9}-\frac{y^2}{4}=0$

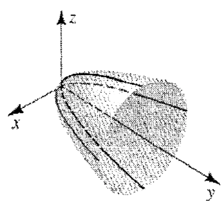
Q. $y^2-\frac{x^2}{4}-z^2=1$

R. $x^2+\frac{y^2}{4}-z^2=1$

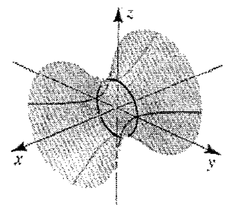
9 K



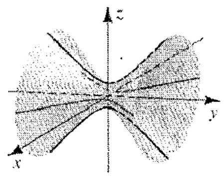
10 N



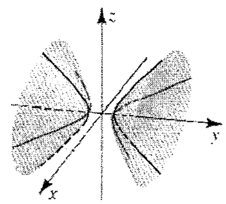
11 C



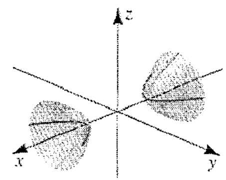
12 O



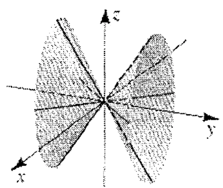
13 Q



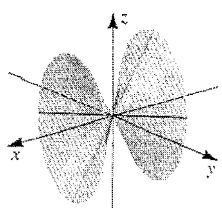
14 H



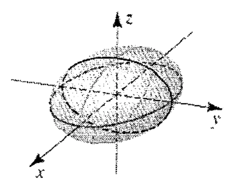
15 P



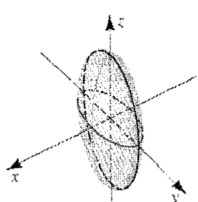
16 G



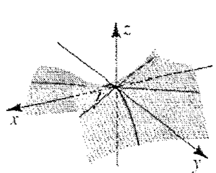
17 A



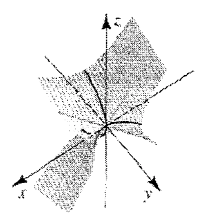
18 J



19 E



20 I



LESSON 3 (12.6)
Cylinders

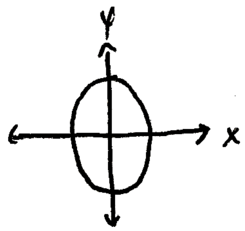
(infinite)

Definition of **Cylinder:** surface that consists of all lines (rulings) that are parallel to a given axis and pass through a plane curve

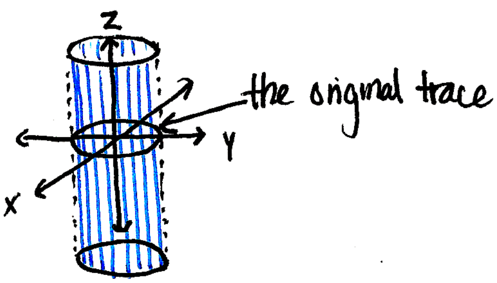
Sketch each graph in $\Re^2$ and $\Re^3$. Identify and describe the surface in $\Re^3$.

* ~~not~~ not a stacking of ellipses

1. $\frac{x^2}{4}+\frac{y^2}{9}=1$ for all z



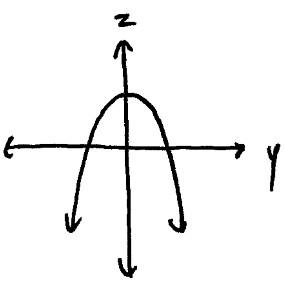
R^2



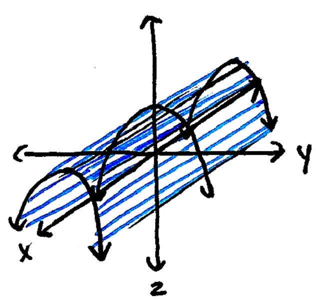
R^3

Desc:
Elliptical cylinder with rulings parallel to z-axis

2. $y^2 = 9 - z$



R^2

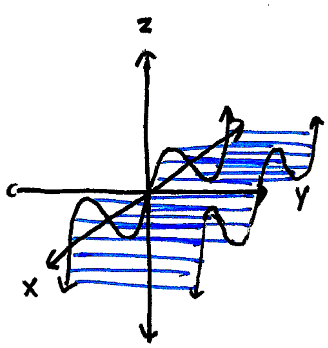


R^3

Desc:
Parabolic cylinder with rulings parallel to x

3. $z = \sin x$

Skip R^2



Desc:
Sinusoidal cylinder with rulings parallel to y

4. $y = 3$ Plane (line cylinder)